Cesare Beccaria: "An Attempt at an Analysis of Smuggling"

Preface

Cesare Bonesana, Marchese di Beccaria (*c.* 1738–1794), was an eighteenth-century economist and sociologist of international repute attained early in his life. Contemporaries linked his name with those of figures like Montesquieu, and his ideas were influential in places as distant as the court of Catherine the Great. After briefly holding the chair of economics in Milan he became an important figure in the Milanese administration. He was an early Utilitarian, and Schumpeter compares his economics lecture notes, written in 1769–1770, favourably with the work of Adam Smith. Along with some friends he founded a society which published a well-reputed scientific and literary journal, *Il Caffè*. It is in this periodical that Beccaria's charming little paper on smuggling first appeared.

The article hardly requires any comment. It may be helpful to the reader to describe briefly the derivation of Beccaria's basic equation $x + tx/u = u$, where u is the total value of the merchandise in question, t is the total tax which should be paid on the whole merchandise, and x is the (unknown) break-even quantity of merchandise which the smuggler must succeed in smuggling. Then t/u is the tax rate per (dollar) of merchandise, and xt/u is the tax avoided by smuggling x dollars worth of merchandise. In order to end up with x dollars worth of goods (valued at cost) the merchant then has two options. He may spend x dollars on the goods and (importing them legally) spend xt/u dollars on duty or, alternatively, he may purchase u dollars worth of merchandise and smuggle it in the expectation that $u - x$ of this amount will be confiscated. The point of zero net advantage is then clearly given by Beccaria's equation. It is this result which Schumpeter describes as "discovery of the idea which underlies modern indifference-variety analysis,"[1] a judgment which seems a bit strong.

It may be of some interest that Beccaria's brief work on this subject (which is reproduced here in its entirety) was some 30 years later carried a bit further by another Italian economist, Guglielmo Silio, in his *Saggio sull' influenza dell' analisi nelle scienze politiche ed economiche*, in Vol. V of *Nuova Raccolta di Opuscoli di Autori Siciliani*, Palermo, 1792, though he did so somewhat

[1] Schumpeter, *History of Economic Analysis*, p. 179.

inaccurately. Silio actually plotted the curve which Beccaria describes verbally in a rather obscure manner. Silio's extensions, which are described in some detail by Theocharis,[2] unfortunately do not seem to be very interesting in themselves.

[2] Reghinos D. Theocharis, *Early Developments in Mathematical Economics*, St. Martin's Press, New York, 1961, pp. 24–27.

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An Attempt at an Analysis of Smuggling

by Cesare Beccaria

Algebra is simply a precise and straightforward technique for reasoning about quantities, and it can therefore be employed not only in geometry and in other mathematical sciences but also in the analysis of anything that is capable of increasing or decreasing, and to all things which exhibit mutally comparable relationships. Even political sciences can therefore make use of algebra, up to some point. Debts and assets of a nation, taxes, etc. are all items which can be treated as quantities and can therefore be subjected to calculation. I said, "up to some point" because political phenomena are highly dependent on many isolated decisions and human passions which cannot be specified precisely. A political system composed of numbers and calculations would be much more suitable to the inhabitants of the island of *Laputa* than to present-day Europeans. Yet, despite the fact that the subject of this article is not very important for the world in general and will be appreciated only by persons of very special inclination, I shall undertake to give an example of the manner in which economic sciences can be approached analytically.

When the government imposes a duty on a merchandise which enters or leaves the country it usually also decrees that merchants who attempt to avoid payment of the duty will lose their merchandise. The risk incurred by the government is then proportional to the amount of the duty, while the merchant's risk is proportional to the value of the merchandise. If the tax is equal to the value of the goods then risks incurred by the two parties are equal. If the tax is greater than the value of the merchandise the government's risk is greater than that incurred by the merchants. Notice, moreover, that the risk incurred by the merchant is directly proportional to the number of customs inspectors and is inversely proportional to the volume of merchandise. These principles are so clear that it would be pedantic to describe them in greater detail. But we can extend this analysis to approach the solution to a problem important for the budget of the State, the determination of means for dealing with the smuggling of certain goods in or out of the country. I repeat that what I have to say does not constitute a solution to the problem, which I have as yet not been able to determine, but can lead towards it.

We shall determine what amount of a certain merchandise must escape taxation by the government before smugglers can avoid any reduction in their capital even if they lost the remainder of the merchandise. Determination of such a quantity will serve as a general guide in the design of tariffs.

Let u be the intrinsic value of the merchandise; t the total tax levied upon it; x the portion of merchandise required [to be smuggled successfully] and d the difference between the tax and the value of the merchandise. The ratio of the total value to the total tax will be equal to the ratio of the required [successfully smuggled] portion to the corresponding tax, that is $u:t = x:\frac{tx}{u}$, where the last term represents the portion of the total tax corresponding to the required value, x.

We have, as the basic condition characterizing the problem, the equation $x + \frac{tx}{u} = u$; multiplying: $ux + tx = uu$, and dividing: $x = \frac{uu}{u + t}$.

But the tax may be equal to the value of the goods, so that $t = u$, or it can be greater than that value by the quantity d, that is $t = u + d$, or less than that value by the same quantity d, that is $t = u - d$.

Substituting these values for t in the general equation $x = \frac{uu}{u + t}$, we obtain the following:

When $t = u$, then

$$x = \frac{uu}{u + u} = \frac{uu}{2u} = \frac{u}{2}.$$

When $t = u + d$, then

$$x = \frac{uu}{u + u + d} = \frac{uu}{2u + d} < \frac{u}{2}.$$

When $t = u - d$, then

$$x = \frac{uu}{u + u - d} = \frac{uu}{2u - d} > \frac{u}{2}.$$

Suppose that in the equation $ux + tx = uu$, we consider t and x to be variables and u to be a constant. The locus of the equation will be a hyperbola between asymptotes whose abscissae, t, taken on one asymptote at a distance u from the asymptotic angle, plus the same distance, will be related to the ordinates x parallel to the other asymptote by a constant ratio, that is, the square of the power u. Inspection of the figure (by those who desire to draw it) will indicate all the different possibilities permitted by the equation.

From these calculations we can derive the following general theorem: given equal volumes, equal intensity of control, and maximal ability on the part of the merchants, the minimal amount of undetected smuggled merchandise required for smuggling to offset the cost of duties is given by the square of the value of the merchandise, divided by the sum of that value and that of the tax.

The utility of these results to someone who is levying tariffs is knowledge on the extent of the smuggling he must still fear even after some number of reprisals.

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